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### The Logarithmic Normal Distribution Function

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LETTER TO THE EDITOR

## The Logarithmic Normal Distribution Function

The generalized form of the logarithmic normal distribution may be written [1]

$$W_s(x) = A_s x^s \exp[-(\ln x - \ln x_s)^2 / 2\sigma_s^2] \quad (1)$$

where  $A_s = 1/\{(2\pi)^{1/2} \sigma_s x_s^{s+1} \exp[(s+1)^2 \sigma_s^2 / 2]\}$  is the normalization constant,  $s$  is a real number called the order of the distribution, and  $x_s$  and  $\sigma_s$  are parameters determining the position and breadth of the distribution. This function seems to have been first introduced into polymer chemistry by Lansing and Kramer in 1935 [2] for the case  $s = 0$ , and later by Wesslau in 1956 [3] for  $s = -1$ .

An extensive examination of the properties of the logarithmic normal distribution was published by Espenscheid et al. [4], who illustrated the different characteristics of curves of different orders. As examples they plotted two families of curves: one for the distribution of order  $s = -1$  (Wesslau) with the same median, i.e.,  $x_{-1}$  value; the other of order  $s = 0$  (Lansing-Kramer), those, however, with the same mode, i.e.,  $x_0$  value. It is clear from their work that they considered that the functions of different orders correspond to different real distributions, and they appear to have been the first workers to propose Eq. (1) (Ref. 4, Eq. 18), calling it the "general family of logarithmically skewed distributions."

If Eq. (1) refers to a distribution over sizes of molecules in a polymer sample, then the variable  $x$  is the degree of polymerization (or molecular weight), and  $W_s(x)$  is the weight function.

The number- and weight-average degrees of polymerization are given by

$$x_n = x_s \exp \{ (2s + 1) \sigma_s^2 / 2 \} \quad (2)$$

and

$$x_w = x_s \exp \{ (2s + 3) \sigma_s^2 / 2 \} \quad (3)$$

from which

$$x_w/x_n = \exp \{ \sigma_s^2 \} \quad (4)$$

Thus the value of the parameter  $\sigma_s$  is independent of the order  $s$ , since it is uniquely determined by the values of the averages  $x_n$  and  $x_w$ . This parameter may therefore be denoted simply by  $\sigma$ .

Suppose now there is another polymer sample having the same average degrees of polymerization  $x_n$  and  $x_w$  but a different logarithmic normal distribution over its molecular sizes, of order  $r \neq s$ , i.e., the distribution

$$W_r(x) = A_r x^r \exp \{ -(\ln x - \ln x_r)^2 / 2\sigma^2 \} \quad (5)$$

The functions are more easily handled after making the straight forward substitution  $y = \ln x$ , such that Eq. (5) becomes

$$W_r(x) = A_r \exp \{ ry \} \exp \{ -(y - y_r)^2 / 2\sigma^2 \} \quad (6)$$

Without the introduction of any assumption we may write

$$W_r(x) = A_r \exp \{ sy \} \exp \{ (r - s)y - (y - y_r)^2 / 2\sigma^2 \} \quad (7)$$

From Eq. (2),

$$\ln x_n = \ln x_s + (s + \frac{1}{2})\sigma^2 = \ln x_r + (r + \frac{1}{2})\sigma^2 \quad (8)$$

i.e.,

$$y_r = y_s + (s - r)\sigma^2 \quad (9)$$

Using this expression for  $y_r$  in Eq. (7) yields

$$\begin{aligned} W_r(x) &= A_r \exp \{ sy \} \exp \{ -(y^2 - 2yy_s + y_s^2) / 2\sigma^2 - (s - r)y_s - (s - r)^2 \sigma^2 / 2 \} \\ &= A_r B \exp \{ sy \} \exp \{ -(y - y_s)^2 / 2\sigma^2 \} \end{aligned} \quad (10)$$

where  $B$  is a constant independent of the variable  $y$ . By using Eq. (9) it can be shown that the relationship between the normalization constants  $A_s$  and  $A_r$  is

$$A_S = BA_T \quad (11)$$

or, in other words, the functions  $W_S(x)$  and  $W_T(x)$  are identical.

There is therefore only one logarithmic normal distribution function.

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