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The Logarithmic Normal Distribution Function

J. G. Watterson[a]
[a] Department of Industrial and Engineering Chemisty Swiss Federal, Institute of Technology Zurich, Switzerland

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LETTER TO THE EDITOR

The Logarithmic Normal Distribution Function

The generalized form of the logarithmic normal distribution may be written [1]

$$W_s(x) = A_s x^s \exp[-(\ln x - \ln x_s)^2/2\sigma_s^2] \tag{1}$$

where $A_s = 1/\{(2\pi)^{1/2} \sigma_s x_s^{s+1} \exp[(s+1)^2\sigma_s^2/2]\}$ is the normalization constant, s is a real number called the order of the distribution, and x_s and σ_s are parameters determining the position and breadth of the distribution. This function seems to have been first introduced into polymer chemistry by Lansing and Kramer in 1935 [2] for the case s = 0, and later by Wesslau in 1956 [3] for s = -1.

An extensive examination of the properties of the logarithmic normal distribution was published by Espenscheid et al. [4], who illustrated the different characteristics of curves of different orders. As examples they plotted two families of curves: one for the distribution of order s = -1 (Wesslau) with the same median, i.e., x_{-1} value; the other of order s = 0 (Lansing-Kramer), those, however, with the same mode, i.e., x_0 value. It is clear from their work that they considered that the functions of different orders correspond to different real distributions, and they appear to have been the first workers to propose Eq. (1) (Ref. 4, Eq. 18), calling it the "general family of logarithmically skewed distributions."

If Eq. (1) refers to a distribution over sizes of molecules in a polymer sample, then the variable x is the degree of polymerization (or molecular weight), and $W_s(x)$ is the weight function.

The number- and weight-average degrees of polymerization are given by

$$x_n = x_s \exp\{(2s+1)\sigma_s^2/2\} \tag{2}$$

and

$$x_w = x_s \exp\{(2s+3)\sigma_s^2/2\} \tag{3}$$

from which

$$x_w/x_n = \exp\{\sigma_s^2\} \quad (4)$$

Thus the value of the parameter σ_s is independent of the order s, since it is uniquely determined by the values of the averages x_n and x_w. This parameter may therefore be denoted simply by σ.

Suppose now there is another polymer sample having the same average degrees of polymerization x_n and x_w but a different logarithmic normal distribution over its molecular sizes, of order $r \neq s$, i.e., the distribution

$$W_r(x) = A_r x^r \exp\{-(\ln x - \ln x_r)^2/2\sigma^2\} \quad (5)$$

The functions are more easily handled after making the straight forward substitution $y = \ln x$, such that Eq. (5) becomes

$$W_r(x) = A_r \exp\{ry\} \exp\{-(y - y_r)^2/2\sigma^2\} \quad (6)$$

Without the introduction of any assumption we may write

$$W_r(x) = A_r \exp\{sy\} \exp\{(r - s)y - (y - y_r)^2/2\sigma^2\} \quad (7)$$

From Eq. (2),

$$\ln x_n = \ln x_s + (s + \frac{1}{2})\sigma^2 = \ln x_r + (r + \frac{1}{2})\sigma^2 \quad (8)$$

i.e.,

$$y_r = y_s + (s - r)\sigma^2 \quad (9)$$

Using this expression for y_r in Eq. (7) yields

$$W_r(x) = A_r \exp\{sy\} \exp\{-(y^2 - 2yy_s + y_s^2)/2\sigma^2 - (s - r)y_s - (s - r)^2\sigma^2/2\}$$

$$= A_r B \exp\{sy\} \exp\{-(y - y_s)^2/2\sigma^2\} \quad (10)$$

where B is a constant independent of the variable y. By using Eq. (9) it can be shown that the relationship between the normalization constants A_s and A_r is

$$A_s = BA_r \qquad (11)$$

or, in other words, the functions $W_s(x)$ and $W_r(x)$ are identical.

There is therefore only one logarithmic normal distribution function.

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J. G. Watterson

Department of Industrial and Engineering Chemisty
Swiss Federal Institute of Technology
Zurich, Switzerland

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